$$\Phi_j^n = \tfrac{1}{2}\left[\left(\frac{x_{j+1}^n - x_j^n}{v_{j+\frac{1}{2}}^n}\right) + \left(\frac{x_j^n - x_{j-1}^n}{v_{j-\frac{1}{2}}^n}\right)\right] \tag{5.3}$$

= average mass at j.

At the left boundary

$$\Phi_o^n = (\tfrac{1}{2})(x_1^n - x_o^n)/v_{\frac{1}{2}}^n \tag{5.4}$$

The new coordinate is given by:

$$x_j^{n+1} = x_j^n + u_j^{n+\frac{1}{2}}\,\Delta t. \tag{5.5}$$

2. Continuity equation:

$$v_{j+\frac{1}{2}}^{n+1} = v_{j+\frac{1}{2}}^n + \Delta t\left(\frac{\rho o}{m}\right)_{j+\frac{1}{2}}\left(u_{j+1}^{n+\frac{1}{2}} - u_j^{n+\frac{1}{2}}\right) \tag{5.6}$$

where

$$m_{j+\frac{1}{2}} = \rho_{j+\frac{1}{2}}^o\left(x_{j+1}^o - x_j^o\right) = \text{mass in the cell } j+\tfrac{1}{2}, \tag{5.7}$$

ρo = initial density.

3. Linear viscosity:

$$q_{j+\frac{1}{2}}^{n+\frac{1}{2}} = C_L\, \rho_{j+\frac{1}{2}}^o\, \eta_{j+\frac{1}{2}}^{n+\frac{1}{2}}\left|u_{j+1}^{n+\frac{1}{2}} - u_j^{n+\frac{1}{2}}\right| \text{ for}\begin{cases} u_{j+1}^{n+\frac{1}{2}} < u_j^{n+\frac{1}{2}} \\ v_{j+\frac{1}{2}}^{n+1} < v_{j+\frac{1}{2}}^n \end{cases} \tag{5.8}$$

= 0 otherwise.

Here

$$\eta_{j+\frac{1}{2}}^{n+\frac{1}{2}} = 2v_o/\left(v_{j+\frac{1}{2}}^{n+1} + v_{j+\frac{1}{2}}^n\right). \tag{5.9}$$

4. Constitutive relations:

The relaxation equation:

$$\alpha^{n+1}_{j+\frac{1}{2}} = \alpha^{n}_{j+\frac{1}{2}} + \left(\frac{\alpha^{eq}-\alpha}{\tau}\right)^{n}_{j+\frac{1}{2}} \Delta t. \qquad (5.10)$$

τ is the characteristic relaxation time and is assumed to be constant.

The specific volume of the first phase is:

$$v^{n+1}_{1,j+\frac{1}{2}} = v^{n+1}_{j+\frac{1}{2}} - (v_2-v_1)\alpha^{n+1}_{j+\frac{1}{2}} \qquad (5.11)$$

Temperature calculation:

$$T^{n+1}_{j+\frac{1}{2}} = T^{n}_{j+\frac{1}{2}} + [CT]^{n}_{j+\frac{1}{2}}(v^{n+1}_{j+\frac{1}{2}}-v^{n}_{j+\frac{1}{2}})$$

$$- \left(\frac{q}{C_v}\right)^{n+\frac{1}{2}}_{j+\frac{1}{2}}(v^{n+1}_{j+\frac{1}{2}}-v^{n}_{j+\frac{1}{2}}) \qquad (5.12)$$

where $C = -(1/C_v)(\partial p/\partial T)_v$ (convenient for a mixed phase)

or $C = -\Gamma/v$ (convenient for a single phase), and Γ is the Grüneisen coefficient. The value of C_v depends on the phase region as described in the last section. The formula for $C_{v,m}$ in a mixed phase is given in Appendix II.

Equation of state:

$$p^{n+1}_{j+\frac{1}{2}} = [p(v_1,T)]^{n+1}_{j+\frac{1}{2}} \qquad (5.13)$$

where $p(v_1,T)$ is given by Eq. (4.5).

The boundary between a single and a two-phase region is distinguished by the transition pressure p_M, at which the relative volume of the first phase is given by v_A.